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# Equilibrium Point Analysis of Synergy Based on Bass Diffusion Model

**Yujian Zhao, Boxi Zuo & Jing Huang**

## Abstract

Based on the Bass diffusion model, this paper analyzes the equilibrium point of synergy, and conducts correlation analysis around the equilibrium point, studies the nature of the equilibrium point and the synergy situation of the equilibrium point, analyzes and proves the stability condition of the equilibrium point and the significance of management and economics behind it, and further clarifies the prerequisite and basis for the synergy subject or element to achieve strategic synergy. Through the study of this paper, the dynamic mechanism of long-term strategic synergy and the spillover effect of synergy are discussed, so that the participants or elements of synergy can cooperate more effectively and produce good benefits or greater value spillover.

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About Author (s)

**Boxi Zuo** (corresponding author), Management School, Jinan University (JNU), Guangzhou, China.
**Yujian Zhao**, Management School, Jinan University (JNU), Guangzhou, China.
**Jing Huang**, Management School, Jinan University (JNU), Guangzhou, China.

## 1. Introduction and Literature Review

Synergy refers to the corresponding changes in organization, structure, and dynamics between two or more systems or elements of a system. It represents a self-developing and improving state or situation that arises from the mutual constraints and promotions among systems or elements within a system. In September 2013, China first proposed the cooperative initiative to build the Belt and Road. The Belt and Road initiative aims to establish sound economic partnerships between China and countries along its routes, achieving mutual benefits, and creating a community of shared future for humanity, providing an important strategic platform (Zhang, n.d.). The transnational network structure of the Belt and Road is a complex, extensive, and hierarchically structured network system of competition and cooperation. Through the collaborative operation between the participants on the network chain, seamless docking of demand, produce, supply and sale can be achieved, so as to improve the welfare level of each participant. In this significant era, the concept of synergy has gained widespread attention in academia and practice, and related issues regarding synergy have become a popular research topic. There is a considerable amount of research, including methodological studies and domain-specific research focusing on synergistic studies. Moreover, with the continuous development and advancement of IT technology, analytical tools, and research concepts, synergistic research continues to develop and deepen.

Currently, there is a growing body of research on synergy theory. Zhang (2019) conducted a knowledge network and graph analysis of regional development synergy using Citespace and Pajek. The study mainly focused on the field of synergy, including synergistic innovation, synergistic development, regional synergy, and industrial synergy. Srivastava and Gnyawali (2011) pointed out that for companies to achieve technological advantage, they must synergize technology with internal resources to make breakthrough innovations. Chen and Zhong (2017) believe that the key to realize the synergy development between provinces and regions is to promote the economic development of backward areas and developed areas, so as to realize the common progress of each other. Therefore, regional synergistic development can be seen as a form of regional economic synergy aimed at promoting economic complementarity and common development among different regions (Sun & Zuo, 2023). Yang and Gu (2011) stated that research indicates that theories on the evolution of industrial structure, regional layout, new institutional economics, and government regulatory mechanisms provide strong theoretical support for the synergistic development of the tertiary industry. Dai and Liu (2016) suggested that the Marxist theory of division of labor and synergy points out a feasible way for us, which not only involves the production development of human society, but also involves changing the traditional production mode and promoting economic progress, so as to achieve the goal of synergy. Mao et al. (2017) proposed that with the continuous development of the Chinese economy, the synergy development of cross-regional industrial ecological economic systems can effectively protect the ecological environment, extend the lifespan of enterprises, enhance their value, and promote rapid economic growth, thereby bringing positive impacts to China. The paper aims to analyze and study the mechanisms of synergy in order to understand the synergistic effects and overflow generated through the organic integration of synergy, rather than simply the accumulation of individual efforts. In this process of integration, specific mechanisms are required to ensure the realization and overflow of synergistic effects. This section analyzes and examines the mechanisms that facilitate constructive synergy, with a particular focus on determining the equilibrium point and analyzing its properties. The research explores the driving mechanisms of long-term strategic synergy and the spillover effects of synergy, aiming to enhance the effectiveness of the participating entities or elements in synergy, and generate greater benefits or value overflow.

## 2. Synergy Mechanism

### 2.1 Bass diffusion model

Bass Diffusion Model proposed by Frank M.Bass and its related extension theory provide an effective market analysis method for enterprises, which can help enterprises better identify and respond to the needs of emerging products and technologies. Accurately estimate the needs of emerging consumer groups in the field of durable goods, so as to achieve effective marketing strategies. Many studies have shown that the development pattern of new methods and new processes can be expressed by Bass's formula (1) :

$$\frac{dN(t)}{d(t)} = p[m - N(t)] + q\frac{N(t)}{m}[m - N(t)] \qquad (1)$$
$$N(t) = mF(t)$$

*m*——The total number of adopters, i.e. the market potential;

*P*——The innovation factor (external influence) is the likelihood that people who have not yet used the product will be influenced by mass media or other external factors to start using the product; Its range is $p \in [0,1]$,That is, it does not affect the sphere of influence of the full replacement;

*q*——Coefficient of imitation (internal influence), that is, the likelihood that people who have not yet used the product will be influenced by the word of mouth of users to start using the product; Its range is $q \in [0.1]$, that is, it does not affect the sphere of influence of the full replacement;

*N(t)*——The cumulative user at time *t*;

*m(t)*—— Represents the number of adopters at time *t*;

*F(t)*——The probability that the number of adopters accounts for the total number of potential adopters at *t*;

*f(t)*——Represents the probability density function of the number of adopters as a percentage of the total number of potential adopters at time *t*;

Make $y(t) = m - N(t)$ , that is $N(t) = m - y(t)$,The difference equation is used to analyze and solve the Bass model. According to the derivation of the function with respect to time, formula (1) can be converted into:

$$-\frac{dy(t)}{dt} = py(t) + q\frac{m - y(t)}{m}y(t) \qquad (2)$$

According to the method and steps of solving Bernoulli equation, the result can be obtained. On the basis of the result of y(t), the expression returned is as follows:

$$\begin{cases} N(t) = \frac{mp[1 - e^{(p+q)t}]}{p + qe^{-(p+q)t}} \\ n(t) = \frac{mp(p+q)^2 e^{(p+q)t}}{[p + qe^{-(p+q)t}]^2} \end{cases} \qquad (3)$$

In equation (3), the coefficients of the "seekers" and "watchers" *q* directly influence the state of the alliance system. If $q > p$, meaning that the internal influence is greater than the external influence, the Bass diffusion curve has a maximum point, indicating that *n(t)* reaches its maximum value. This result suggests that the business alliance possesses a certain level of influence, capable of attracting external companies to have the willingness to join, thereby maximizing the expansion of the alliance participants at a given moment. If $q \le p$, meaning that the external influence is not less than the internal influence, the diffusion curve does not have an extremum point. Over time, *n(t)* exhibits an exponential decay, indicating that the alliance has lower attractiveness to external companies and struggles to gain new allies. There are no extremum points or maximum points for *n(t)*, and it may eventually lead to the termination or dissolution of the alliance.

### 2.2 Symbolic description of the model

In order not to lose the generality and the typicality of research and analysis, the mathematical model is modeled and analyzed based on the synergy of two participants.

$k$: The potential stock of all resources for collaborative participants, $k > 0$;
$m(t)$ :The existing resource stock of the participant A at time $t$, $m(t) > 0$ ;
$n(t)$ :The existing resource stock of the participant B at time $t$, $n(t) > 0$ ;
$\Delta m(t)$ Is the total resource value spillover amount of *Ra* after the cooperation of participants at time t. When the value spillover is negative, participants will launch collaborative alliance. Therefore, this paper considers the positive value spillover of synergy, so $\Delta m(t) > 0$.
$\Delta n(t)$ Is the total resource value spillover amount of *Rb* after participating in the main body synergy at time t. Based on the premise that synergy is mutually beneficial, this paper considers the positive value spillover of synergy. So $\Delta n(t) > 0$.
$\delta$ Represents the influence coefficient of *Rb*'s resource stock in *CAs* of the synergy alliance on *Ra*'s synergy spillover value after the cooperation of participants, $\delta > 0$. The larger $\delta$ gets, the greater the effect of *Rb* on *Ra*, the worse the synergistic effect. And vice versa.
β represents the influence coefficient of *Ra*'s resource stock in *CAs* of the collaborative alliance on *Rb*'s synergistic spillover value after the cooperation of participants $\beta > 0$. The greater the $\beta$ value, the greater the effect of *Ra* on *Rb,* the greater the influence on the pair and the worse the synergy effect; and vice versa.
The spillover effect mainly depends on the synergistic effect between the elements $\sigma$, $\beta$ , therefore, its range is $(-\infty, +\infty)$.

**2.3 The mechanism model of synergy**

According to the Bass diffusion model, the spillover value of the synergy within *CAs* can be divided into two parts. One part is the external spillover value, which is related to the "comparative advantage effect" and "economies of scale effect", that is, after the synergy, participants can better exert their own advantages, further exert their comparative advantages, and at the same time exert their scale advantages. Form external spillover value. The other part is the internal spillover value, which is related to the "learning effect". After the participants cooperate, they can focus more on their own advantages, produce learning effect, and constantly improve efficiency. According to the difference of the overflow value channel, the overflow value model of the regional synergy in *CAs* is built. Suppose *Ra*,*Rb*∈CAs are the two participants in synergy. There is spillover demand of synergistic economic growth among those two.

(1) Participant *Ra*
In time period $\Delta t$, Suppose the external spillover value obtained by *Ra* participants in CAs after synergy is $\Delta m$, $\Delta m$ is consists of two parts. One part is the value increment of "external synergy spillover", that is, due to synergy, considering the value increment generated by external "comparative advantage" and "economies of scale effect":

$$p_1[k\text{-}m(t)\text{-}\delta n(t)]\Delta t \quad (4)$$

The parameter $\delta$ is due to the external of the two co-subject value spilt, but there is also a negative impact on the other part of the subject. The other part is the value increment of "internal synergy overflow", because the "learning effect" is an internal value overflow that is generated by the increase in the utilization of the resource utilization:

$$q_1[k - m(t) - \delta n(t)]m\Delta t \quad (5)$$

Formula (4) and (5) are combined to obtain the value spillover increment brought by the participant *Ra* after *CAs* synergy: $\Delta m = p_1[k\text{-}m(t)\text{-}\delta n(t)]\Delta t + q_1[k - m(t) - \delta n(t)]m\Delta t$

Merge up: $\Delta m = [k\text{-}m(t)\text{-}\delta n(t)][p_1 + q_1 m(t)]\Delta t$ (6)

If we make $\Delta t \to 0$ than we can turn formula (6) into :

$$\frac{dm}{dt} = [k\text{-}m(t)\text{-}\delta n(t)][p_1 + q_1 m(t)] \quad (7)$$

(2) Participant *Rb*

Similarly, it can be obtained that the increment of internal value spillover brought by *Rb* participating in $t$ due to synergy is: $p_2[k - n(t) - \beta m(t)]\Delta t$. Similarly, the external value spillover increment is: $q_2[k\text{-}n(t)\text{-}\beta m(t)]n\Delta t$ ,

Then there is $\Delta n = [k - n(t) - \beta m(t)][p_2 + q_2 n(t)]\Delta t$ (8)

Compared with external synerovalue overflow, the internal synergy overflow value increment is a multifactor nowing, because the value increment of internal synerosed overflow and the volume of the body's own resources are also related to the system. If we make $\Delta t \to 0$,we can turn formula (8)into :

$$\frac{dn}{dt} = [k\text{-}n(t)\text{-}\beta m(t)][p_2 + q_2 n(t)] \quad (9)$$

By synthesizing equations (7) and (9), the differential equations of resource spillover value after *Ra* and *Rb* synergy in *CAs* can be obtained as follows:

$$\begin{cases} \frac{dm}{dt} = [k - m(t) - \delta n(t)][p_1 + q_1 m(t)] \\ \frac{dn}{dt} = [k - n(t) - \beta m(t)][p_2 + q_2 n(t)] \end{cases} \quad (10)$$

The coefficients $p_1$、$q_1$、$p_2$、$q_2$、$\delta$、$\beta$ are all greater than zero. In formula (10), the equilibrium point must be satisfied $\frac{dm}{dt} = 0$、$\frac{dn}{dt} = 0$ Analyze the second factor, we can get:

$$\begin{cases} p_1 + q_1 m = 0 \\ p_2 + q_2 n = 0 \end{cases} \quad (11)$$

That is $m = -\frac{p_1}{q_1}$ , $n = -\frac{p_2}{q_2}$. The solution results are located in the third quadrant, because $p_1$、$q_1$、$p_2$、$q_2$、$m$、$n$ are all greater than zero, so this equilibrium point is meaningless, and other equilibrium points are discussed in detail below.

## 3. The Equilibrium Point Analysis of Synergy

Synergy hopes to achieve "play comparative advantage", "produce scale effect", "produce learning effect", and such synergy can reach a long-term strategic alliance. Whether it can be stable for a long time is mainly the problem of analyzing the equilibrium point of synergy, that is, whether the equilibrium point of synergy exists and whether the existing equilibrium point is stable.

### 3.1Class of equilibrium point

For a system of second-order homogeneous linear differential equations: $\frac{dx}{dt} = AX$,

among: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in R$

Definition 1: $p = -(a_{11} + a_{22}); q = detA$

Criterion 1: In the case of different values, the equilibrium point has different properties, as shown in Table 1, which is also the criterion for determining the properties of the equilibrium node.

**Table 1: Criterion for determining the properties of the equilibrium node**

| Serial number | $p = -(a_{11} + a_{22})$ , $q = \det A$ | Equilibrium type | Stability |
|---|---|---|---|
| 1 | $p > 0,\ q > 0,\ p^2 > 4q$ | Stable node | Stable |
| 2 | $p > 0,\ q > 0,\ p^2 = 4q$ | Stable degenerate node | Stable |
| 3 | $p < 0,\ q > 0,\ p^2 > 4q$ | Unstable node | Unstable |
| 4 | $p < 0,\ q > 0,\ p^2 = 4q$ | Unstable degenerate node | Unstable |
| 5 | $q \le 0$ | Saddle point | Unstable |

(1) Stable node
If the equilibrium point is satisfied at a certain point $o = (0,0)$, and satisfies$p > 0, q < 0, p^2 > 4q$ if the orbit is parabolic, then the equilibrium point is called a stable node.
(2) Stable degenerate nodes
If the equilibrium point is satisfied at a certain point$o = (0,0)$, and satisfies$p > 0, q < 0, p^2 = 4q$, if the orbit is parabolic, it is called a stable degenerate node.
(3) Unstable node
If the equilibrium point is satisfied at a certain point$o = (0,0)$, and satisfies$p < 0, q > 0, p^2 > 4q$, if the orbit is parabolic, it is called a unstable node.
(4) Unstable degenerate node
If the equilibrium point is satisfied at a certain point$o = (0,0)$, and satisfies$p < 0, q > 0, p^2 = 4q$, if the orbit is parabolic, it is called a Unstable degenerate node.
(5) Saddle point
If the equilibrium point is satisfied at a certain point$o = (0,0)$and satisfy$q \le 0$ , if the orbit is parabolic, then the equilibrium point is called the saddle point.

**3.2Calculation and discussion of equilibrium point**

The balance point represented by the following factors is analyzed above. In Formula (10), since all variables are non-negative, this balance point has no practical significance and value. Now analyze another set of factors and set them equal to zero to find other equilibrium points, namely:

$$\begin{cases} k - m - \delta n = 0 \\ k - n - \beta m = 0 \end{cases} \tag{12}$$

Taking *m* and *n* as variables, solving the equations of equation (12), we can get:

$$\begin{cases} m = \dfrac{(1-\delta)k}{1-\delta\beta} \\ n = \dfrac{(1-\beta)k}{1-\delta\beta} \end{cases} \begin{cases} m = \dfrac{(1-\delta)k}{1-\delta\beta} \\ n = \dfrac{(1-\beta)k}{1-\delta\beta} \end{cases} \tag{13}$$

If equation (12) is viewed as two linear equations, then the first linear equation $k - m - \delta n = 0$ passes through points $(k, 0)$ and $(0, \frac{k}{\delta})$, and the second linear equation $k - n - \beta m = 0$ passes through points $(0, k)$ and $(\frac{k}{\beta}, 0)$. In view of (13), several cases are discussed.
When $\beta > 1, \delta < 1$
Because$\beta > 1$, so$k > \frac{k}{\beta}$, at the same time$1 - \beta < 0$. In the same way because$\delta < 1$, so$k < \frac{k}{\delta}$, at the same time$1 - \delta > 0$. If$\delta\beta > 1$, then$1 - \delta\beta > 1$, in equation (13)$m < 0$，$n > 0$, that is, the equilibrium point is located in the second quadrant. Similarly, if$\delta\beta < 1$, then$1 - \delta\beta > 1$, in equation (13)$m > 0$，$n < 0$, that is, the equilibrium point is located in the fourth quadrant.
In summary, no matter what the case, the balance point is not in quadrant I, and it has no meaning in real synergy. As in the case of $\delta\beta = 1$ discussed earlier, let's further analyze Figure

6.
In fact, for the equation $k-m-\delta n=0$ of a straight line outside, within the range expressed by the horizontal line, there is $k-m-\delta n>0$, that is, there is $\frac{dm}{dt}>0$. Therefore, for the synergistic participant *Ra*, the internal spillover value of its synergy is constantly increasing. For equation $k-m-\beta m=0$ with a straight line inside, within the range expressed by the vertical line, there is $k-m-\beta m>0$, that is, there is $\frac{dn}{dt}>0$. At this time, for the synergistic participant *Rb*, the internal spillover value of its synergy is constantly increasing. After comprehensive analysis, the result of synergy is that the spillover value of the participant *Ra* grows faster than that of *Rb* synergy, because the first line in equation (13) is located outside the second line, that is, in the middle area of the two lines (non-public interval), $\frac{dn}{dt}<0$, which means that it will inhibit the growth of spillover value of region *Rb*. The end result of this inhibition is that the role of participant *Rb* is completely overwhelmed by participant *Ra*. If two enterprises cooperate, *Rb* will be merged by *Ra*. If the two regions cooperate, *Rb* will be integrated by *Ra*. If the two elements work together, *Rb* is completely replaced by *Ra*.
The practical management revelation and significance is that $\beta>1,\delta<1$ means that Ra has a large spillover influence coefficient on *Rb*, while *Rb* has a small spillover influence coefficient on *Ra*. Therefore, although $\delta\beta>1$ has an increased overall spillover influence, it means that *Ra* has a larger influence coefficient on Rb, while $\delta\beta<1$ means that the spillover influence is reduced. In both cases, there can be no benign synergy.
When $\delta<1,\beta<1$
Because of $\beta<1$, so $k<\frac{k}{\beta}$, at the same time $1-\beta>0$. In the same way because $\delta<1$, so $k<\frac{k}{\delta}$, and $1-\delta>0$. Two lines have an intersection point, the following further analysis for this intersection point. Because $\delta\beta<1$, then $1-\delta\beta>0$, in equation (13), $m>0$, $n>0$, therefore, the equilibrium point is in quadrant I, which has specific significance in real synergy.
Therefore, at some time $t_M$, in the region of horizontal lines, the slope of the first line in equation (12) is small, because $m(t)<\frac{k-\delta k}{1-\delta\beta}$ and $\frac{dm}{dt}>0$, $m(t)$ gradually increases and tends to the equilibrium point $\frac{k-\delta k}{1-\delta\beta}$. Meanwhile, in this region, because $n(t)>\frac{k-\beta k}{1-\delta\beta}$, the slope of the second line in equation (12) is large, $m(t)$ gradually decreases and tends to the equilibrium point $\frac{k-\beta k}{1-\delta\beta}$. At time $t_M$, in the region of the vertical line, because $m(t)>\frac{k-\delta k}{1-\delta\beta}$, and $\frac{dm}{dt}<0$, $m(t)$ gradually decreases and approaches the equilibrium point $\frac{k-\delta k}{1-\delta\beta}$. At the same time, in this region, because $n(t)<\frac{k-\beta k}{1-\delta\beta}$, therefore $\frac{dn}{dt}>0$, $n(t)$ gradually increases and tends to the equilibrium point $\frac{k-\beta k}{1-\delta\beta}$.

**Theorem 1:** When $\delta<1,\beta<1$ ,the equilibrium point $\begin{cases} m=\frac{(1-\delta)k}{1-\delta\beta} \\ n=\frac{(1-\beta)k}{1-\delta\beta} \end{cases}$ is a stable equilibrium point.

**Proof:** Let function $f(m,n), g(m,n)$ be:

$$f(m,n)=\frac{dm(t)}{dt}=p_1[k-m(t)-\delta n(t)]+q_1 m(t)[k-m(t)-\delta n(t)]$$

$$g(m,n)=\frac{dn(t)}{dt}=p_2[k-n(t)-\beta m(t)]+q_2 n(t)[k-n(t)-\beta m(t)]$$

According to the rule of determining fixed point of equilibrium point, the judgment matrix is constructed:

$$A=\begin{bmatrix} f_m & f_n \\ g_m & g_n \end{bmatrix}=\begin{bmatrix} \frac{\partial f}{\partial m} & \frac{\partial f}{\partial n} \\ \frac{\partial g}{\partial m} & \frac{\partial g}{\partial n} \end{bmatrix} \qquad (14)$$

Where: $\frac{\partial f}{\partial m}=-p_1+q_1k-2q_1m(t)-q_1\delta n(t)=-p_1+q_1[k-m(t)-\delta n(t)]-q_1m(t)$

Because of $k-m(t)-\delta n(t)=0$, there is:

$$\frac{\partial f}{\partial m}=-p_1-q_1m(t),\ \frac{\partial f}{\partial n}=-p_1\delta-q_1\delta m(t),\ \frac{\partial g}{\partial m}=-p_2\beta-q_2\beta n(t),$$

$$\frac{\partial g}{\partial n}=-p_2+q_2k-2q_2n(t)-q_2\beta m(t)=-p_2+q_2[k-n(t)-\beta m(t)]-q_2n(t)$$

Because of $k-n(t)-\beta m(t)=0$, so $\frac{\partial g}{\partial n}=-p_2-q_2n(t)$

So, $A=\begin{bmatrix} f_m & f_n \\ g_m & g_n \end{bmatrix}=\begin{bmatrix} -(p_1+q_1m) & -(p_1\delta+q_1\delta m) \\ -(p_2\beta+q_2\beta n) & -(p_2+q_2n) \end{bmatrix}$ (15)

And $p=-(f_m+g_n)=-[-(p_1+q_1m)-(p_2+q_2n)]=(p_1+p_2)+(q_1m+q_2n)$

Because of $p_1,p_2,q_1,q_2>0$, so $p_1+p_2>0$ .And $\delta<1$, $\beta<1$, so $m>0,n>0$, then $q_1m+q_2n>0$

So, $p=(p_1+p_2)+(q_1m+q_2n)>0$ (16)

$q=\det(A)=f_mg_n-f_ng_m$

$f_mg_n=[-(p_1+q_1m)]\times[-(p_2+q_2n)]=p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn$

$f_ng_m=[-(p\delta_1+q_1\delta m)]\times[-(\beta p_2+\beta q_2n)]=p_1p_2\delta\beta+p_1q_2\delta\beta n+p_2q_1\delta\beta m+q_1q_2\delta\beta mn$

Then

$q=\det(A)=p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn-p_1p_2\delta\beta-p_1q_2\delta\beta n-p_2q_1\delta\beta m-q_1q_2\delta\beta mn$

By combining the terms of the same kind and simplifying, we get: $q=p_1p_2(1-\delta\beta)+p_1q_2n(1-\delta\beta)+p_2q_1m(1-\delta\beta)+q_1q_2mn(1-\delta\beta)$

$=(1-\delta\beta)(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)$

And because $\delta<1$,so $\delta\beta<1$, therefore $1-\delta\beta>0$. Because$\delta<1,\beta<1$,

Therefore $m>0$, $n>0$

Because of $p_1,p_2,q_1,q_2>0$, we get $p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn>0$

$q=(1-\delta\beta)(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)>0$ that is $q>0$ (17)

$p^2-4q=[p_1+p_2+q_1m+q_2n]^2-4(1-\delta\beta)(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)$

$=p^2{}_1+2p_1p_2+p^2{}_2+q_1{}^2m^2+2q_1q_2mn+q_2{}^2n^2+2p_1q_1m+2p_1q_2n+2p_2q_1m$

$+2p_2q_2n-4p_1p_2-4q_1q_2mn-4p_1q_2n-4p_2q_1m+4\delta\beta(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)$

$=p^2{}_1-2p_1p_2+p^2{}_2+q_1{}^2m^2-2q_1q_2mn+q_2{}^2n^2+2p_1q_1m+2p_1q_2n+2p_2q_1m$

$+2p_2q_2n_2-2p_1q_2n-2p_2q_1m+4\delta\beta(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)$

$=[p_1-p_2+q_1m-q_2n]^2+4\delta\beta(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)$

Because$[p_1-p_2+q_1m-q_2n]^2\geq 0;\delta,\ \beta,\ m,n,p_1,p_2,q_1,q_2>0$

Therefor $p^2-4q=[p_1-p_2+q_1m-q_2n]^2+4\delta\beta(p_1p_2+p_1q_2n+p_2q_1m+q_1q_2mn)>0$

That is $p^2-4q_1>0$ (18)

Equations (16), (17) and (18) can be obtained that the equilibrium point is the fixed point. Taking the above situation into account, it can be seen that in such a structure, any small fluctuation in the equilibrium state will return to the equilibrium state. Because of the cooperation at this time, for the participants in the cooperation, the cooperation can obtain relatively good spillover value, and they are willing to cooperate with each other for a long time, forming a stable negative feedback situation. If it is a collaborative case, it reflects that the two elements are just in the best state of input.

Practical management enlightenment and significance: Because the participants Ra and Rb have little influence on each other, they can obtain positive and stable synergistic spillover effect from each other, so they can cooperate and win-win for a long time, but this is a weak synergistic win-win state.

When $\delta > 1, \beta > 1$

Because of $\beta > 1$, so $k < \frac{k}{\beta}$, at the same time $1-\beta < 0$. In the same way because $\delta >$, so $k < \frac{k}{\delta}$, and $1-\delta < 0$. Two lines have an intersection point, the following further analysis for this intersection point.

Therefore, at a certain time $t_M$, the slope of the first line in equation (4-1) in the horizontal line region is large, and because $m(t) > \frac{k-\delta k}{1-\delta\beta}$, and $\frac{dm}{dt} > 0$, $m(t)$ gradually increases and moves away from the equilibrium point $\frac{k-\delta k}{1-\delta\beta}$. Meanwhile, in this region, the slope of the second line in equation (4-1) is small, so $n(t) \to 0$. At time $t_M$, in the vertical region, because $m(t) \to 0$. At the same time, in this region, because $n(t) > \frac{k-\beta k}{1-\delta\beta}$, therefore $\frac{dn}{dt} > 0$, $n(t)$ gradually increases away from the equilibrium $\frac{k-\beta k}{1-\delta\beta}$.

**Theorem 2:** When $\delta > 1, \beta > 1$, the equilibrium point $\begin{cases} m = \frac{(1-\delta)k}{1-\delta\beta} \\ n = \frac{(1-\beta)k}{1-\delta\beta} \end{cases}$ is an unstable equilibrium point.

**Proof:** As before, the judgment matrix $A = \begin{bmatrix} f_m & f_n \\ g_m & g_n \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial m} & \frac{\partial f}{\partial n} \\ \frac{\partial g}{\partial m} & \frac{\partial g}{\partial n} \end{bmatrix}$ of the equilibrium stability rule is constructed (19)

Also like the proof of theorem 1, construct $f_m, f_n, g_m, g_n$ function

$p = -(f_m + g_n) = \text{-[-}(p_1 + q_1 m) - (p_2 + q_2 n)] = (p_1 + p_2) + (q_1 m + q_2 n)$

Because $p_1, p_2, q_1, q_2 > 0$, so $p_1 + p_2 > 0$, again $\delta > 1$, $\beta > 1$, d$m > 0, n > 0$

So $p = (p_1 + p_2) + (q_1 m + q_2 n) > 0$ is $p > 0$ (20).

$q = \det(A) = f_m g_n - f_n g_m$

$$q = p_1 p_2 (1-\delta\beta) + p_1 q_2 n (1-\delta\beta) + p_2 q_1 m (1-\delta\beta) + q_1 q_2 mn (1-\delta\beta)$$
$$= (1-\delta\beta)(p_1 p_2 + p_1 q_2 n + p_2 q_1 m + q_1 q_2 mn)$$

Because of $\delta > 1$, $\beta > 1$, so $\delta\beta > 1$,

Because of $\delta > 1$, $\beta > 1$, so $m > 0$, n $> 0$

Because of $p_1, p_2, q_1, q_2 > 0$ so $q = (1-\delta\beta)(p_1 p_2 + p_1 q_2 n + p_2 q_1 m + q_1 q_2 mn) < 0$, that is $q < 0$ (21)

$p^2 - 4q = [p_1 \text{-} p_2 + q_1 m \text{-} q_2 n]^2 + 4\delta\beta(p_1 p_2 + p_1 q_2 n + p_2 q_1 m + q_1 q_2 mn)$

because of $[p_1 - p_2 + q_1 m - q_2 n]^2 \geq 0$, $[p_1 - p_2 + q_1 m - q_2 n]^2 \geq 0$

So $[p_1 \text{-} p_2 + q_1 m \text{-} q_2 n]^2 + 4\delta\beta(p_1 p_2 + p_1 q_2 n + p_2 q_1 m + q_1 q_2 mn) > 0$

Then $[p_1\text{-}p_2 + q_1m\text{-}q_2n]^2 + 4\delta\beta(p_1p_2 + p_1q_2n + p_2q_1m + q_1q_2mn) > 0$ , and therefore $p^2 - 4q_1 > 0$
From equations (20), (21) and (22), it can be concluded that the equilibrium point is an unstable node. The analysis and summary of the above situations are shown in Table 2. It can be concluded that in such a structure, any small fluctuation in the equilibrium state will push away from the equilibrium state. This is because the synergy at this time has a large spillover value and obvious spillover effect for the participants, but such spillover is based on the premise that it infringes the interests of the other party. Therefore, the amplified value spillover will deepen the willingness of each other not to cooperate and form a stable positive feedback situation, that is, when there is a little unfavorable situation, the participants will not cooperate. It will continue to strengthen and amplify this adverse factor, that is, those with large spillover value will become larger, and those with small spillover value will become smaller, thus losing the power of cooperation, and thus destroying cooperation. For the synergy of elements, there will be a local substitution effect, leading to the weakening of the value of the replaced elements, and eventually forming an unstable state.

**Table 2: Synergy equilibrium point**

| Num | Parameters | | Quadrant | Synergy situation |
|---|---|---|---|---|
| 1 | $\delta<1$ | $\delta\beta>1$ | II | No synergistic significance |
| | $\beta>1$ | $\delta\beta<1$ | IV | No synergistic significance |
| | | $\delta\beta=1$ | I | *Ra* grows faster than *Rb* synergy spillover value, and *Rb* will be completely replaced by *Ra*, resulting in *Rb* not actively participating in synergy |
| 2 | $\delta>1$ | $\delta\beta>1$ | IV | No synergistic significance |
| | $\beta<1$ | $\delta\beta<1$ | II | No synergistic significance |
| | | $\delta\beta=1$ | I | *Rb* has a faster growth in synergistic spillover value than *Ra*, and *Ra* will be completely replaced by *Rb*, resulting in *Ra* not actively participating in synergy |
| 3 | $\delta<1,\beta<1$ | | I | Stable synergy equilibrium point, the best synergy state |
| 4 | $\delta>1,\beta>1$ | | I | Unstable synergistic equilibrium, synergistic effect of local substitution |

Practical management enlightenment and significance: collaborative participants Ra and Rb have a great impact on each other. If the synergy spillover effect is very obvious in the short term, but it is unstable in the long term, this is a short-term win-win situation of strong synergy.

## 4 Conclusion

The paper analyzes and studies the equilibrium point of synergy based on the Bass diffusion model. It concludes that in the presence of competitive participants, synergy tends to have poorer effects and limited duration. Over time, differences in regional value overflow lead to a tendency for homogenization of synergy, resulting in assimilation or mergers among participating entities. If the growth of synergistic value overflow among participants reaches an equilibrium point but their synergistic development directions differ, such synergy becomes unstable. Only when the overflow value from synergy among participants grows positively, can long-term synergy of strategic alliance be formed. This promotes output growth collectively, rather than a zero-sum game leading to imbalanced states. These findings offer theoretical guidance for achieving constructive synergies.

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